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Coulomb drag in the quantum Hall $\nu = \frac{1}{2}$ state: Role of disorder

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Abstract

We consider Coulomb drag between two layers of two-dimensional electron gases subject to a strong magnetic field, with the Landau level filling factor in each layer being $\frac{1}{2}$. We find ρ_D to be *very large*, as compared to the zero magnetic field case. We attribute this enhancement to the slow decay of density fluctuations in a strong magnetic field. For a clean system, the linear q-dependence of the longitudinal conductivity, characteristic of the $\nu = 1/2$ state, leads a unique temperature dependence—$\rho_D \propto T^{4/3}$. Within a semiclassical approximation, disorder leads to a decrease of the transresistivity as compared with the clean case, and a temperature dependence of $T^2 \log T$ at low temperatures.

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I. INTRODUCTION

In this paper we consider the transresistivity of a system of two identical parallel two-dimensional electronic layers in close proximity. A current I_1 is driven in one layer, while the current in the other layer is kept zero. Due to the interactions between the electrons at the different layers, a voltage V_2 develops in the second layer. For a square sample, the transresistivity is $\rho_D = -V_2/I_1$. First discussed twenty years ago [1], transresistivity is a subject of recent study, both experimental and theoretical [2–5], and in particular when the system is placed in a perpendicular magnetic field [6,7].

Here, we consider the transresistivity in a strong magnetic field, with the Landau-level filling fractions of the two layers being $\nu_1 = \nu_2 = \frac{1}{2}$. We have considered this problem in detail in [8], neglecting the role of disorder. Here we focus on the latter. Due to length restrictions the discussion and references below are rather brief. We refer the reader to our previous work for a more elaborate discussion.

An experimental study of Coulomb drag in the regime of a partially filled Landau level was recently carried out by Eisenstein, Pfeiffer and West [9]. Two theoretical works [10] considered the problem from a composite-fermion approach.

II. COULOMB DRAG BETWEEN CLEAN $\nu = \frac{1}{2}$ LAYERS

If the layers are sufficiently close, the main contribution to the transresistivity comes from the Coulomb interactions between the electrons. In the limit of weak interlayer coupling, both in the absence and in the presence of magnetic field, the transresistivity is given by [4]

$$\rho_D = \frac{1}{8\pi^2}\frac{h}{e^2}\frac{1}{Tn^2}\int\frac{d\mathbf{q}}{(2\pi)^2}\int_0^\infty \frac{\hbar\, d\omega}{\sinh^2\frac{\hbar\omega}{2T}}\, q^2\left[\mathrm{Im}\Pi(\mathbf{q},\omega)\right]^2 |U_{\rm sc}(\mathbf{q},\omega)|^2 \,. \tag{1}$$

Here $U_{\rm sc}$ is the screened interlayer Coulomb interaction, Π is the single layer density-density response function (irreducible with respect to the Coulomb interaction), n is the average electron density in each layer and T is the temperature. Coulomb drag is a result of scattering events between electrons of different layers, transferring momentum $\hbar\mathbf{q}$ and energy $\hbar\omega$ between the layers. Thus, it reflects the response of the two layers at finite wavevector $\mathbf{q}$ and frequency ω.

In addition to appearing explicitly in Eq. (1), the response function Π also determines the screening of the interlayer interaction. Interaction is described in terms of 2×2 matrices (denoted by hats), with indices corresponding to the two layers. The screened interlayer interaction $U_{\rm sc}$ is the off-diagonal element of

$$\hat{V}_{\rm sc} = \hat{V}_{\rm bare}\,(1 + \hat{\Pi}\hat{V}_{\rm bare})^{-1}\,, \tag{2}$$

where

$$\hat{V}_{\rm bare} = \frac{2\pi e^2}{\epsilon q}\begin{pmatrix} 1 & e^{-qd} \\ e^{-qd} & 1 \end{pmatrix} \quad \text{and} \quad \hat{\Pi} = \begin{pmatrix} \Pi & 0 \\ 0 & \Pi \end{pmatrix}\,. \tag{3}$$

The transresistivity is determined by regions of $\mathbf{q}$ and ω where the integrand of Eq. (1) is large. It is thus interesting to look at the poles of the integrand. They are the solutions of

$$i\omega - \frac{q^2}{e^2}\sigma(\mathbf{q},\omega)(V_b(q) \pm U_b(q)) = 0\,, \tag{4}$$

where Π was expressed in terms of the conductivity σ for physical clarification [8]. The solutions of (4) are the dispersion relations for the decay of symmetric and antisymmetric charge density modulations. In the limit of small q they are given by $i\omega \propto q\sigma(q,\omega)$ for the symmetric modulation, and $i\omega \propto q^2\sigma(q,\omega)$ for the antisymmetric modulation.

For $\nu = 1/2$ the calculation of Π is carried out in the composite-fermion picture for the half-filled Landau level [11]. Response functions are 2×2 matrices (denoted by tildes), with entries corresponding to the density (denoted by 0) and to the transverse current (denoted by 1). The electronic response function $\tilde{\Pi}^e$ is expressed as

$$(\tilde{\Pi}^e)^{-1} = \begin{pmatrix} 0 & \frac{2\pi i\hbar\tilde{\phi}}{q} \\ -\frac{2\pi i\hbar\tilde{\phi}}{q} & 0 \end{pmatrix} + (\tilde{\Pi}^{\mathrm{CF}})^{-1}\,. \tag{5}$$

The matrix appearing explicitly in (5) is the Chern-Simons interaction matrix, with $\tilde{\phi} = 2$ for $\nu = \frac{1}{2}$. The composite-fermion response function $\tilde{\Pi}^{\mathrm{CF}}$ describes the response of the composite fermions to the total scalar and vector potentials, including external, Coulomb and Chern-Simons contributions. The electronic density-density response function appearing in Eqs. (1)–(3) is $\Pi = \Pi^e_{00}$.

Within the random phase approximation, in the limit of $q \ll k_F$ and $\omega \ll qv_F$, we have [11]

$$\Pi(\mathbf{q},\omega) = \frac{q^3}{q^3\left(\frac{dn}{d\mu}\right)^{-1} - 2\pi i\hbar\tilde{\phi}^2\omega k_F}\,, \tag{6}$$

where $dn/d\mu$ is the thermodynamical electronic compressibility of the system. This special form of Π has several consequences. First, it leads to the linear dependence on q of the conductivity, $\sigma(q) \propto q$. As a result, antisymmetric charge density modulations have a very slow decay rate, with $i\omega \propto q^3$, that eventually leads to a large transresistivity. Second, the integrand of Eq. (1) is dominated by q and ω along the line $\omega \propto q^3$, limited by $\omega \approx T$. Hence the important contribution comes from $q \approx q_0(T) = k_F(T/T_0)^{1/3}$, and not $q \approx d^{-1}$ as in the $B = 0$ case (T_0, typically ≈ 190°K, is defined below). Finally, due to these considerations the temperature dependence of the transresistivity is $\rho_D \propto T^{4/3}$. The leading temperature dependence of ρ_D is

$$\rho_D = 0.825\frac{h}{e^2}\left(\frac{T}{T_0}\right)^{4/3} + \mathcal{O}(T^{5/3})\,, \tag{7}$$

where

$$T_0 = \frac{4\pi e^2 nd}{\tilde{\phi}^2\epsilon}(1+\alpha)\,, \quad \alpha = \frac{dn}{d\mu}\frac{2\pi e^2 d}{\epsilon}\,. \tag{8}$$

Typically α is a small parameter. The calculation sketched above is described in detail in [8].

III. SEMICLASSICAL APPROXIMATION FOR DISORDER

The use of Eq. (6) is valid, in the presence of disorder, for $q \gg l_{\rm el}^{-1}$, where $l_{\rm el}$ is the single-layer composite fermion elastic mean free path. This puts a limit on the validity of the calculations sketched above [8], which may be expressed in terms of the single layer longitudinal resistivity as

$$\rho_{xx} \ll 2\frac{h}{e^2}\left(\frac{T}{T_0}\right)^{1/3} . \tag{9}$$

For typical experimental values this condition is barely fulfilled, as $\rho_{xx} \approx 2500\ \Omega$, while the right hand side of Eq. (9) is approximately 4000–8000 Ω.

To include disorder, we first use the Boltzmann equation to find the composite-fermion response function $\tilde{\Pi}^{\rm CF}$ [12,13]. The Boltzmann equation for the composite fermion distribution function $f(\mathbf{r},\mathbf{k},t)$ in the presence of an electric field $\mathbf{E}$ is given by

$$\frac{\partial f}{\partial t} + \frac{\partial f}{\partial \mathbf{r}}\cdot\frac{\hbar\mathbf{k}}{m^*} - \frac{1}{\hbar}\frac{\partial f}{\partial \mathbf{k}}\cdot e\mathbf{E} = \left(\frac{\partial f}{\partial t}\right)_{\rm sc} , \tag{10}$$

where m^* is the composite fermion effective mass. With $\mathbf{E}(\mathbf{r},t) = \mathbf{E}\exp(i\mathbf{q}\cdot\mathbf{r} - i\omega t)$, one seeks a solution of the form

$$f(\mathbf{r},\mathbf{k},t) = f_0(\epsilon_{\mathbf{k}}) + \left(-\frac{\partial f_0}{\partial \epsilon}(\epsilon_{\mathbf{k}})\right) f(\phi)\exp(i\mathbf{q}\cdot\mathbf{r} - i\omega t) , \tag{11}$$

where ϕ is the polar angle in the $\mathbf{k}$ plane, and $f_0(\epsilon)$ is the equilibrium Fermi-Dirac distribution function. The scattering term is taken here in the relaxation time approximation (see comments below)

$$\left(\frac{\partial f(\phi)}{\partial t}\right)_{\rm sc} = -\frac{f(\phi) - \bar{f}}{\tau} , \quad \text{where} \quad \bar{f} = \frac{1}{2\pi}\int_0^{2\pi} d\phi\, f(\phi) . \tag{12}$$

Here, τ is the composite-fermion transport mean free time.

Solving Eq. (10), f is used to obtain the current in the system, using

$$\mathbf{j}(\mathbf{r},t) = -e\int \frac{d\mathbf{k}}{(2\pi)^2}\frac{\hbar\mathbf{k}}{m^*} f(\mathbf{r},\mathbf{k},t) , \tag{13}$$

from which the conductivity, and hence the response functions, may be extracted. The result is

$$\Pi_{00}^{\rm CF} = \frac{m^*}{2\pi\hbar^2}\left(1 + \frac{i\omega}{\sqrt{(1/\tau - i\omega)^2 + (qv_{\scriptscriptstyle F})^2} - 1/\tau}\right) , \tag{14a}$$

$$\Pi_{11}^{\rm CF} = -\frac{q^2}{24\pi m^*} - i\frac{\omega}{q^2}\frac{m^*}{2\pi\hbar^2}\left(\frac{1}{\tau} - i\omega - \sqrt{(1/\tau - i\omega)^2 + (qv_{\scriptscriptstyle F})^2}\right) . \tag{14b}$$

The diamagnetic term in $\Pi_{11}^{\rm CF}$ does not come out of semiclassical calculation, and is added by hand. The electronic response function Π is found using (5). It has the correct form, Eq.

(6), in the clean limit ($\tau \to \infty$). In the diffusive limit ($\omega\tau \ll 1$ and $ql \ll 1$) we obtain the diffusive response function

$$\Pi = \frac{dn}{d\mu}\frac{D_e q^2}{-i\omega + D_e q^2}\,, \quad \text{with} \qquad D_e = \frac{1+\tilde{\phi}^2/12}{1+(\tilde{\phi}k_F l/2)^2}\frac{v_F^2\tau}{2}\,. \tag{15}$$

The simplest form of relaxation time approximation, Eq. (12), may not be appropriate for the dynamics of composite fermions [11,13]. However, a diffusive motion is described by a density–density response function of the form (15). Thus, we expect an improved relaxation time approximation to affect the value of D_e, but not the functional form of (15).

For low enough temperatures ($q_0(T)l \ll 1$) the transresistivity is dominated by the diffusive regime, and is given by (cf. [4,5])

$$\rho_D = \frac{2\pi}{3}\frac{h}{e^2}\left(\frac{T}{T_{\text{dif}}}\right)^2 \ln\left(\frac{T_{\text{dif}}}{2T}\right)\frac{1}{(k_F d)^4}\,, \quad T_{\text{dif}} = \frac{4\pi e^2}{\epsilon d}\frac{dn}{d\mu}\frac{D_e}{\hbar}\,. \tag{16}$$

Eq. (15) may also describe a diffusive motion at zero magnetic field, where the diffusion constant is usually much larger than D_e. Indeed, at zero magnetic field and fantastically low temperatures a $T^2 \log T$ dependence of ρ_D was predicted in [4]. The smallness of the diffusion constant in a strong magnetic field, as well as the emphasis on small q's in the clean limit, makes the $T^2 \log T$ regime in the $\nu = 1/2$ case hold at experimentally accessible temperatures.

To obtain the transresistivity in the regime of interest, interpolating the clean and disordered limit, the full expression (14) for $\tilde{\Pi}^{\text{CF}}$ must be used. Eq. (5) is then used to obtain Π, Eq. (2) to obtain U_{sc}. The transresistivity is then given by Eq. (1). The integration in Eq. (1) is carried out numerically. Fig. 1 shows the transresistivity for different values of disorder. The parameters used in the calculation are $n = 1.4 \times 10^{11}$ cm^{-2}, $d = 200$ Å, and $m = 4m_b$ where m_b is the bare mass of the electron in GaAs. The amount of disorder included is moderate: At $T = 0.5°$K we have $q_0 l = 8.0$, 3.2, and 1.6 for $\rho_{xx} = 1000\ \Omega$, 2500 Ω, and 5000 Ω. But, as can be seen in the figure, disorder reduces the transresistivity rather significantly—by a factor of order 2. The transresistivity remains much larger (by 3–4 orders of magnitude) than at $B = 0$.

IV. SUMMARY

In this paper we consider the Coulomb drag between two layers of two-dimensional electron gases at Landau level filling fractions of $\nu_1 = \nu_2 = \frac{1}{2}$. The coupling between the layers is discussed in purely electronic terms. We find

- Coulomb drag at $\nu = 1/2$ is *much larger* than in the absence of a magnetic field. The enhancement is due to the slow decay of density fluctuations in a strong magnetic field.

- For the clean case, we find that a unique temperature dependence of the transresistivity—$\rho_D \propto T^{4/3}$.

- At $\nu = 1/2$ disorder reduces Coulomb drag (as opposed to its role in the absence of a magnetic field), leading to a $T^2 \log T$ dependence at low temperatures. In typical experimental values the effect of disorder on ρ_D is not negligible.

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FIGURES

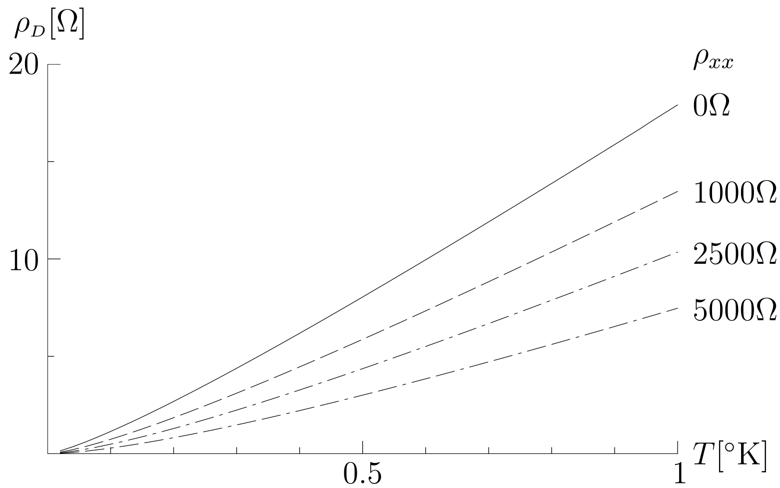

FIG. 1. The transresistivity as a function of temperature, for different values of disorder. Details are given in the text.